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A "PERFECT" HYDRODYNAMIC SIMILARITY AND THE EFFECT OF SMALL-SCALE VORTICES ON THE LARGE-SCALE DYNAMICS

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Abstract

In the laboratory experiments designed to reproduce hydrodynamical phenomena of relevance for astrophysics the Reynolds numbers, although very large, are usually smaller than in real astrophysical systems. If the hydrodynamic flow reaches the turbulent state, it may then happen that differences (related to the difference in Reynolds numbers) would appear in the global-scale motions of the two systems. The difficulty in studying this issue in high energy density laboratory experiments lies in that equations of state and transport coefficients are usually not very well known, so that the subtle effect of the Reynolds number may be easily obscured by experimental uncertainties. An approach has recently been suggested [D.D. Ryutov, B.A. Remington, Phys. Plasmas, ***10****, 2629, 2003] that allows one to circumvent this difficulty and isolate the effect of the Reynolds number. In the present paper, after presenting a summary of the previous results, we briefly discuss various aspects of possible experiments.*

I. INTRODUCTION

Hydrodynamic flows that we encounter in astrophysics (e.g., supernova explosions, astrophysical jets, accretion discs, etc) have usually very high Reynolds numbers (just because of very large spatial scales involved). Accordingly, laboratory experiments designed to simulate these phenomena (see reviews [1,2]) should also have large Reynolds numbers. Large Reynolds number flows mean that viscous dissipation does not play a role in the global-scale motion, which can then be adequately described by equations of ideal hydrodynamics (Euler equations).

There exist a convenient scaling [3-5] which relates the systems described by equations of the ideal magnetohydrodynamics (MHD); for example, this scaling relates a real astrophysical system and its laboratory counterpart, which can be 10 to 20 orders of magnitude smaller in size. However, it is very difficult to make the "laboratory" Reynolds number, although very large, the same as the "astrophysical" Reynolds number. It then becomes unclear whether the two systems would evolve similarly if both reach a state of highly developed turbulence, where the vortices on dissipative scales would appear. A question that is most important in this context is a question on whether two systems with different (large) Reynolds numbers (say, 10^6 and 10^7) and identical in all other respects would behave differently on the global scale. [It should be noted that the dissipative-scale vortices are quite small, well below the resolution of any imaging system (see Sec. 3 for some numerical examples).]

A possible approach to answering this question might be studying the evolution of two systems which are scalable to each other in the ideal hydrodynamics but would have different Reynolds numbers. Then, the differences in their behavior (other than associated with the scaling transformations) would be a measure of the dissipative effects. However, applying the scalings from Refs. [3-5] for isolating subtle dissipative effects is difficult, because the equations of state (EOS) in the regimes typical for high-energy density (HED) hydrodynamic experiments are usually known relatively poorly. Also poorly known are dissipative coefficients, like kinematic viscosity and thermal diffusivity. Therefore, a difference in the behavior of the two systems, if present, could be attributed also to the uncertainties caused by a poor knowledge of EOS and dissipative coefficients, and the main question would remain unanswered.

It has recently been pointed out [6] that there is a very simple similarity, called "perfect similarity" in [6], that can in principle be used for answering this intriguing question. In our present paper, after a brief description of the "perfect similarity" we consider energy requirements for HED experiments where this scaling could be used, and possible impact of such factors as heat loss, irreproducibility of experimental data, and molecular mix.

II. PERFECT SIMILARITY

Consider a non-dissipative MHD, without making any assumptions with regard to the equation of state, and allowing for a possibility of spatial variation of the chemical composition.

The similarity simply consists of transforming $\boldsymbol{r}$ and t to

$$\boldsymbol{r}' = A\boldsymbol{r}; \quad t' = At, \tag{1}$$

where A is a constant scaling factor, and leaving $\mathbf{v}$, $\boldsymbol{B}$, p, and ρ unchanged. The set of equations for ideal MHD is:

$$\begin{gathered} \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \\ \rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p - \frac{1}{4\pi}\mathbf{B}\times\nabla\times\mathbf{B} \\ \frac{\partial \mathbf{B}}{\partial t} = \nabla\times\mathbf{v}\times\mathbf{B} \end{gathered} \tag{2}$$

where $\mathbf{v}$, ρ, p and $\mathbf{B}$ are the velocity, the density, the pressure, and the magnetic field, respectively (CGS units are used). We have to supplement these equations with the energy equation, which reads as:

$$\frac{\partial \varepsilon}{\partial t} + \mathbf{v}\cdot\nabla\varepsilon = -(\varepsilon + p)\nabla\cdot\mathbf{v} \tag{3}$$

where $\varepsilon = \varepsilon\,(p,\rho, C)$ is the internal energy per unit volume. This equation implies that there are no dissipative processes in the fluid, so that the entropy of any fluid element remains constant. The parameter C is used to characterize a fluid with a varying composition. If one deals with a more than a 2-component fluid, one can introduce several such parameters. As, in the framework of the ideal hydrodynamics, the mutual diffusion of various materials is negligible, one has:

$$\frac{\partial C}{\partial t} + \mathbf{v}\cdot\nabla C = 0. \tag{4}$$

Let us now make a transition from the initial system, described by Eqs. (2)-(4), to a "primed" system, in which Eq. (1) holds , and

$$\mathbf{v}' = \mathbf{v}, \quad \mathbf{B}' = \mathbf{B}, \quad \rho' = \rho, \quad p' = p, \quad C' = C. \tag{5}$$

Substituting $\boldsymbol{r}'/A$, t'/A, $\mathbf{v}'$, $\boldsymbol{B}'$, ρ', p', and C' for $\boldsymbol{r}$, t, $\mathbf{v}$, $\boldsymbol{B}$, ρ, p, and C in Eqs. (2)-(4), one finds that the primed equations are identical to the un-primed ones. As an example, we present the primed version of Eq. (3):

$$\frac{\partial \varepsilon(p',\rho',C')}{\partial t'} + \mathbf{v}'\cdot\nabla'\varepsilon(p',\rho',C') = -\left[\varepsilon(p',\rho',C') + p'\right]\nabla'\cdot\mathbf{v}' \tag{6}$$

which is indeed identical to Eq. (3). The same can be easily checked for the other equations thereby proving similarity. One can check (Cf. Ref. [5]) that the shock boundary conditions are also invariant.

No approximations are involved, whence the suggested term "perfect similarity." So, all the differences in the behavior of two systems related to each other by this similarity would be a direct measure of the role of non-ideal effects.

Obviously, the drive in the primed system (say, an ablation pressure p_{abl} on the ablation surface) must be the same in the two systems, being only subject to similarity transform (1),

$$p'_{abl}(\mathbf{r}', t') = p_{abl}(\mathbf{r}'/A, t'/A) \tag{7}$$

The energy required for driving a scaled experiment is obviously

$$W' = A^3 W, \tag{8}$$

An important merit of the perfect similarity is that it does not change transport coefficients (e.g., shear kinematic viscosity ν) which remain equal at the corresponding points of the initial and primed systems (this is because the viscosity ν is a function of p, ρ, and C, which are equal in the corresponding points of the two systems). Accordingly, the Reynolds number, $\mathrm{Re} = L\mathrm{v}/\nu$, scales as

$$\mathrm{Re}' = A\,\mathrm{Re} \tag{9}$$

The same is true for the Peclet number, $Pe = L\mathrm{v}/\chi$, the Peclet mass number, $Pe_m = L\mathrm{v}/D$, and the magnetic Reynolds number, $\mathrm{Re}_M = L\mathrm{v}/D_M$ (where χ, D, and D_M are thermal diffusivity, inter-spieces diffusion coefficient, and magnetic diffusivity):

$$Pe' = APe; \; Pe'_m = APe_m; \quad \mathrm{Re}'_m = A\,\mathrm{Re}_m \tag{10}$$

So, the perfect similarity holds for an arbitrary equation of state; for an arbitrary varying composition; and in the presence of shocks.

To be specific, we will refer to a small scale system as an "unprimed" system, and to a large-scale system as to a "primed" system; in other words, we assume that A>1.

III. ASSESSING THE EFFECT OF SMALL-SCALE VORTICES ON THE GLOBAL SCALE MOTION

As an illustration, we discuss the possibility of applying the perfect similarity approach to an experiment on the development of the Richtmyer-Meshkov (RM) and Rayleygh-Taylor (RT) instability of the bump on the interface of two different materials, as shown on Fig. 1. To evaluate the feasibility of such an experiment, we use, as a reference point, two recent exemplary experiments by H. Robey et al [7, 8] devoted to the studies of various aspects of the RM-RT instabilities. These experiments were carried out with the Omega laser, with the use of ten beams and the total laser energy delivered to the target ~ 5 kJ.

A deeply nonlinear evolution of pre-imposed perturbation on the interface between the plastic and CH foam was studied. The characteristic diameter of the package d was ~ 800 μm. It was enclosed in a tube (in most cases, Beryllium) to delay the propagation of a shock around the sides of the target. The characteristic wavelength λ of perturbations was ~ 50-70 μm. An accuracy of measuring geometrical dimensions of the Rayleigh-Taylor structures was in the range of 5% -10%.

diameter from ~ 800 mm, would allow one to reduce the required drive energy by a factor of ~ 8, i.e., to approximately 0.6 kJ.

Assuming that the energy available at the NIF for driving the scaled-up target of Fig. 1 will be 0.9 MJ (out of the total 1.8 MJ), one finds that the scaling factor A will be in this case ~15. As the target will be much thicker, a smaller amount of tracer will have to be added or, alternatively, a more penetrating backlighter has to be used.

The Reynolds number Re in experiments [7, 8] was of order of 10^5 (see Fig. 7 of Ref. [8]). It will be approximately the same in the experiment with the axisymmetric dimple mentioned above. In the scaled experiment with the NIF, the Reynolds number will be 15 times higher. For sometimes assumed logarithmic dependence of the global scale motion on the Reynolds number (e.g., [9]), changing the latter from 10^5 to $1.5{\cdot}10^6$ would mean a change of order of 15-20% in the global-scale motion. This would already be distinguishable. A power-law dependence of the global-scale dynamics on the Reynolds number would be easier to find. In this case, there will be no need in changing geometrical dimensions by a factor of 10 (requiring a thousand-fold change in the

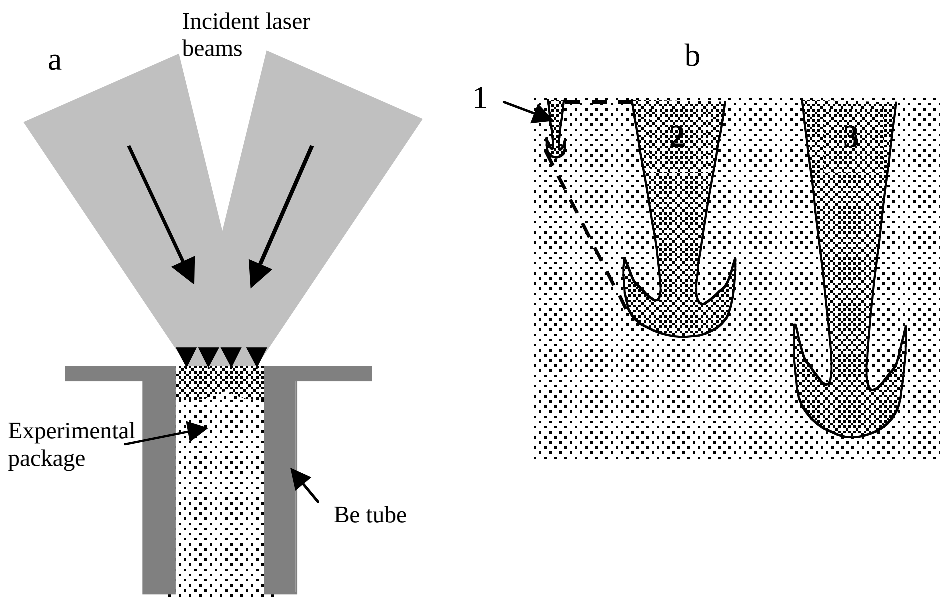

Fig. 1. Possible experiment on the study of the effect of the Reynolds number on the Rayleigh-Taylor instability: a) General experimental setup, with the experimental package accelerated by the ablation pressure (shown in short arrows); the lightly hatched material is denser then the heavily hatched; initial perturbation shown as a bump in the denser material, will evolve into familiar "mushroom" on the nonlinear stage. The difference between two experiments would be the geometric dimensions and the pulse duration; b) Rough sketch of strongly non-linear Rayleigh-Taylor "bubble" originating from the initial bump in a small-scale experiment (1), in a "perfectly similar" larger-scale experiment in the case of weak Reynolds number effects (2), and in the case of substantial Reynolds number effects (3); the structure (2) is geometrically similar to the structure (1), whereas the structure (3) is expected to be longer in the same instance of time as structure (2). We do not show small-scale vortices excited in the interface zone which can be smeared by them.

As reduction of the drive energy is of much importance for the perfect similarity approach (see Eq. (8)), it is beneficial to study the evolution of a single axisymmetric bump (dimple), as shown on Fig. 1, not a multi-wave perturbation as in Refs. [7, 8]. Taking the radius of the dimple r_0 to be 70 μm, and leaving a 70 μm gap between the edge of the dimple and the beryllium tube, one finds that the inner diameter of the tube can be made equal to d~280 μm. This reduction of the tube deposited energy) — a moderate change in the scale by a factor of 1.5 – 2 would be sufficient to detect the power-law dependence at the aforementioned level of accuracy.

Within the Kolmogorov-Obukhov model (e.g., [10]), the scale of dissipative vortices l_{diss} is related to the global scale L by:

$$l_{diss} \sim L(\mathrm{Re}_{\mathrm{crit}}/\mathrm{Re})^{3/4}, \qquad (11)$$

where $\mathrm{Re}_{\mathrm{crit}}$ is the critical Reynolds number for the onset of the instability of the shear flow (e.g., [10]). This is a parameter much greater than 1, between 100 and 1000 depending of the type of the motion. Assuming $\mathrm{Re}_{\mathrm{crit}} \sim 300$, one finds $l_{diss} \sim 50 L \mathrm{Re}^{-3/4}$. Taking as a global scale the dimple radius r_0, one finds that the dissipative scale is $l_{diss} \sim 5 \cdot 10^{-5}$ cm in the "small" system and $l_{diss} \sim 10^{-4}$ cm in the "large" system. These scales are certainly well below the resolution of existing imaging techniques. On the other hand, their effect (if present) on the global-scale motion would be discernable even for a relatively weak logarithmic dependence of the global flow on the Reynolds number.

Other candidate experimental configurations include pulsed jets [11], flows past the body [12], and turbulent mix experiments [14].

IV. NON-STEADY-STATE EFFECTS

In the systems of the type shown on Fig. 1, the shear-flow turbulence is driven by an "external" (in this case, the RT) instability. Such systems are inherently non-steady-state, and the turbulence has to respond to the changing external conditions. Without getting into any details of this complex problem, which is still an area of active research (e.g., [14, 15]), we just mention that, if the dissipative-scale vortices have not been formed yet, both systems should evolve according to equations of the ideal hydrodynamics. In this case, there will be no difference between the primed and unprimed systems, aside from the difference in scales (see a caveat regarding possible probabilistic effects later in this section). The time required for the vortices at the dissipation scale to develop depends on the Reynolds number. Therefore, if the difference between the outcomes of the two experiments is present, this may mean that one of them has reached the state of developed turbulence, whereas the other has not.

The development of small-scale vortices may depend on whether the transition between two materials is smooth, with a gradual variation of density, or sharp, with a step-wise density variation (see Ref. [3] for more detail). In the latter case, the small-scale perturbations are driven at an early stage directly by the RT instability, whereas at the former case they are not. In astrophysical systems the transition between various layers is typically relatively smooth. However, such targets are more complex in manufacturing. The selection between these two types of targets would require some further analysis.

It was tacitly assumed in our analysis that the large-scale motion is perfectly reproducible in both the initial and primed systems, up to possible inaccuracies in the manufacturing the targets. On the other hand, it may happen that the turbulence would have a "bursty" behavior at the relatively large scales, still resolvable in the images of the global flow. For the Reynolds numbers in question, such features by themselves must be much bigger than the dissipative scale (11). As the bursts may have a statistical character, they would lead to shot-to-shot irreproducibility of the images and make a direct comparison between the primed and unprimed systems impossible.

The presence of bursts can be checked in the experiments with the unprimed (small-scale) system. Taking several shots with the identical initial conditions and comparing the images of the flow at the same instants of time, one would be able to detect the bursty behavior. If these bursts are present and appear in a statistical fashion (which may or may not be the case), one would have to resort to a statistical approach, by making a several "identical" shots in both primed and unprimed systems, averaging the images, and comparing the results for the averaged images. This would, of course, be more costly than in the case where the global flow is deterministic.

V. OTHER DISSIPATIVE PROCESSES

Thus far, we have been concentrating on the viscous processes. They indeed are critical in establishing dynamical properties of the flow. However, the thermal diffusivity is normally substantially higher than the kinematic viscosity, so that the thermal diffusivity becomes important at larger sizes of the vortices than the viscosity. On the other hand, in order the thermal conduction to play a role in the dynamical evolution of the system, the compressibility effects must be important. So, if one wants to isolate viscous effects, one has to look into the settings where compressibility plays a subdominant role. This is the case where the motion on this smaller scales is strongly subsonic, which it usually is.

There are special situations (delineated in Ref. 16) where thermal conductivity leads to the onset of new, dissipative, instabilities in the systems that are otherwise RT stable. Clearly, on should avoid such situations if the *Re* effect is the main target of the experiment.

In the system with a varying composition, a mutual diffusion of the species characterized by the mass Peclet number Pe_m may become important. It would lead to a molecular mix when the vortices reach the smallest scales. If there is a concern that this effect may obscure viscous effects, one may switch to packages made of one single material, for example, the foam of a varying density.

VI. DISCUSSION

As we have shown, the “perfect similarity” approach allows evaluating effect of the Reynolds number on the global scale motion in HED experiments. This can be done despite substantial uncertainties with EOS and transport coefficients, and our inability to resolve small-scale vortices, provided the geometrical scales of large-scale features are measured with a sufficient accuracy. A concept of a dedicated experiment has been proposed and it has been pointed out that meaningful results can be obtained with ~ 1-10 kJ of energy on target. Outstanding possibilities will open up when NIF beams reach ~100 kJ of energy on target (1st cluster) and improve even further for full NIF (~1 MJ on target); another good candidate could be Z [17]. The experiment discussed would serve as a direct discovery tool, not just a tool for code benchmarking: we do not know at present (and will hardly know in a few years to come) what dependence on the Reynolds number comes out of such an experiment (but this only adds fun and suspense to the whole undertaking!)

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